FORMALIZATION OF FAIR PLAY STRATEGIES FOR EPISTEMIC GAMES IN STRATEGY LOGIC

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Context

- Winning : not the only goal of a game (learning, storytelling)
- Our goal : formalize complex strategies not based on winning
- Fair play : a strategy that is not only based on winning
- Focus : formalization of fair play strategies using logic

Example

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Simplified game of the President

- 52 cards divided equally among players
- Winning condition : a player discards all their cards
- Round begins : current player can play :
  - Single card
  - Pair of cards
  - Triple of cards
  - Quadruple of cards
- Subsequent players play the same type of hand with value above the previous (i.e. you can only play pairs during a round if the first hand was a pair)
- Round ends when no player can play

Example

Alice plays along a strategy

- Bob desires to play a pair of 7 and plays after Alice
- Alice starts the round, she plays a pair of 6 when she could have played a single 6

Is this strategy fair play ?

- Intuitively **yes** because Bob will be able to play what he wanted.
- But if Alice did not know Bob's intentions, she would have been just playing for her, meaning it would have been **not particularly fair play**
- If Alice knew Bob's intentions and would have played a single 6, it would be **not fair play**

Outline

1. Introduction
2. Modeling strategies in literature
3. Formalization of fair play strategies
4. Conclusion

Outline

Strategy Logic (Mogavero et al., 2010)

Strategy : Function $\sigma : H \to \mathcal{A}$ where H : set of histories and $\mathcal{A}$: set of actions.
Strategy logic operators :

- All standard logic operators $\neg, \wedge, \vee, \Rightarrow, \Leftrightarrow$
- LTL operators $X, F, G, U \ldots$
- Quantifiers over strategies $\langle\langle x \rangle\rangle, [[x]]$
- Binding operators (a, x)

Example of a formula in Strategy Logic :

$$\mathcal{G}, \chi, w \models \langle\langle x \rangle\rangle (a, x) [[y]] (b, y) F \text{wins}_a$$

Formalization of mental states (Adam et al., 2009)

Modal operators are used to reason about the mental states of the agents in a game.

- $K_a(\varphi)$: the agent a knows φ
- $\text{Des}_a(\varphi)$: the agent a desires φ
- $\text{Prob}_a(\varphi)$: φ is plausible for the agent a, i.e. for a, φ is true in a majority of possible worlds

We note with a hat (e.g. $\widehat{\text{Des}}_a(\varphi)$) the dual of these operators, e.g. $\widehat{\text{Des}}_a(\varphi)$ means φ is compatible with the desires of a.

Outline

Concurrent game structure model

A concurrent game structure is defined by a tuple
$\mathcal{G} = (N, \Phi, \mathcal{A}, W, \bar{w}, T, \{\sim_a\}_{a \in N}, \{\mathscr{D}_a\}_{a \in N}, \{\mathscr{P}_a\}_{a \in N}, U, g, V)$ where:

- N : set of players, Φ : set of atomic propositions, $\mathcal{A}$: set of actions
- W : set of states, $\bar{w}$: initial state, T : set of terminal states
- $\sim_a, \mathscr{D}_a, \mathscr{P}_a$: semantics for K_a, Des_a, Prob_a for each player $a \in N$
- $U : W \times \prod_{i \in N} A^i \to W$: update function
- $g : N \to 2^W$: function that give the winning states for each agents
- $V : W \to 2^\Phi$: valuation function

Non-blocking fair play strategies - Example

Non-blocking strategy : Strategy where the player does not block its opponent

Example :

- Alice knows Bob desires to play a pair of 7 and plays after Alice
- Alice starts the round, she plays a pair of 6
- Alice could have played a single 6

What makes this move fair play ?

- Alice has the **knowledge** of Bob's **desires**
- **Alice's strategy** does **not block** Bob's desires
- There was an **alternative strategy** that would have blocked Bob's desires
- Doing this strategy does **not go against her desires**

Non-blocking strategies - Formalization

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x is a non-blocking strategy of a towards b

$$
\begin{aligned}
\mathcal{G}, \chi, w \models\, & K_a(\mathsf{Des}_b(\varphi)) \\
& \wedge \langle\langle y \rangle\rangle (b, y)(a, x)\varphi \\
& \wedge \langle\langle x' \rangle\rangle (a, x')[[y]](b, y)\neg\varphi \\
& \wedge \widehat{\mathsf{Des}}_a(\varphi)
\end{aligned}
$$

Non-blocking strategies - Limitations

New scenario :

- Now 3 players : Alice, Bob and Charles
- Alice wants to be non-blocking for Charles
- Alice has a double 6, Bob has a single 8, Charles has a double 9
- Charles wants to play at least a card
- Alice **knows** everyone's game and Charles' **desires**
- Charles thinks Alice will play a double 6

Playing a single 6 or a double 6 for Alice **are both non-blocking for Charles**
Playing a single 6 however would **surprise** Charles because it was the most **plausible move for him**

Non-surprising strategies - Example

Non-surprising strategy : Strategy where the player plays along what is the most plausible move for its opponent

Example :

- Alice knows Charles thinks Alice will play a pair of 6
- Alice plays a pair of 6

What makes this move fair play ?

- Alice **knows** what Charles **think will happen**
- She plays **according to Charles' assumptions**
- She does not play **against her desires**

Non-surprising strategies - Formalization

x is a non-surprising strategy of a towards b

$$\mathcal{G}, \chi, w \models K_a\big(\neg K_b(\varphi) \wedge \mathsf{Prob}_b(\varphi)\big)$$
$$\wedge \left[(a,x)\langle\langle y\rangle\rangle(b,y)\varphi \vee \left(\begin{array}{c}[[x']](a,x')[[y]](b,y)\neg\varphi \\ \wedge \\ (a,x)[[y]](b,y)K_b(\neg\varphi)\end{array}\right)\right]$$
$$\wedge \widehat{\mathsf{Des}}_a(\varphi)$$

Outline

Conclusion

Recap :

- Strategy Logic for formalizing strategies
- Modal operators to formalize mental states
- Non-blocking strategies formalization not to block the opponent's desires
- Non-surprising strategies formalization to play according the opponent's assumptions about our moves

Next steps :

- Test formalization on hand made strategies
- Implement President and test on real play
- Synthesize fair-play strategies
- Extending the formalization to more than two players

The end

[thank you]

Any Questions ?

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